Lakatos, Piroska
Salem numbers defined by Coxeter transformation. (English) Zbl 1281.11093
Linear Algebra Appl. 432, No. 1, 144-154 (2010).

A generalised star is a simple connected graph that is not a tree, not a single cycle and has exactly one vertex of degree at least 3. Such a graph is made into an oriented n-vertex graph \( Q \) by giving each edge a unique orientation, but not allowing oriented cycles. Its adjacency matrix \( B \) is then an \( n \times n \) matrix that has a 1 if there is an edge from \( i \) to \( j \), and 0 otherwise. The Coxeter transformation \( c_\mathcal{Q} \) of \( Q \) is then the matrix \( \Phi_\mathcal{Q} = -(E - B)^{-1}(E - B)^T \), where \( E \) is the \( n \times n \) identity matrix. This is an integer matrix, since the ‘no oriented cycles’ condition implies that the vertices of \( Q \) can be labelled so that \( B \) is strictly upper-triangular. The characteristic polynomial of \( Q \) is its Coxeter polynomial. The main result of this paper is that this Coxeter polynomial is the product of a Salem number and certain cyclotomic polynomials. Thus the spectral radii of such polynomials are Salem numbers. The proof is a detailed analysis of the roots of the Chebyshev transform (trace polynomial) of the Coxeter polynomial.

Remark: At the bottom of page 145 ‘\( a_{2n} = 0 \)’ should read ‘\( a_{2n} \neq 0 \)’

Reviewer: Chris Smyth (Edinburgh)

MSC:

11R06 PV-numbers and generalizations; other special algebraic numbers; Mahler measure
11C08 Polynomials in number theory
05C50 Graphs and linear algebra (matrices, eigenvalues, etc.)
20F55 Reflection and Coxeter groups (group-theoretic aspects)
30C15 Zeros of polynomials, rational functions, and other analytic functions of one complex variable (e.g., zeros of functions with bounded Dirichlet integral)

Keywords:
Coxeter polynomial; Salem number; semi-reciprocal polynomials; Chebyshev transform

Full Text: DOI

References:


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