Sun, Yidong; Ma, Luping

Minors of a class of Riordan arrays related to weighted partial Motzkin paths. (English)
Zbl 1284.05036

Summary: A partial Motzkin path is a path from \((0,0)\) to \((n,k)\) in the \(XOY\)-plane that does not go below the \(X\)-axis and consists of up steps 
\(U = (1,1)\), down steps \(D = (1,-1)\) and horizontal steps \(H = (1,0)\).
A weighted partial Motzkin path is a partial Motzkin path with the weight assignment that all up steps and down steps are weighted by 1, the horizontal steps are endowed with a weight \(x\) if they are lying on \(X\)-axis, and endowed with a weight \(y\) if they are not lying on \(X\)-axis. Denote by \(M_{n,k}(x,y)\) the weight function of all weighted partial Motzkin paths from \((0,0)\) to \((n,k)\), and 
\[M_{n,k} = (M_{n,k}(x,y))_{n \geq k \geq 0}\] the infinite lower triangular matrices.

In this paper, we consider the sums of minors of second order of the matrix \(M\), and obtain a lot of interesting determinant identities related to \(M\), which are proved by bijections using weighted partial Motzkin paths. When the weight parameters \((x,y)\) are specialized, several new identities are obtained related to some classical sequences involving Catalan numbers. Besides, in the alternating cases we also give some new explicit formulas for Catalan numbers.

MSC:
05A19 Combinatorial identities, bijective combinatorics
11B65 Binomial coefficients; factorials; \(q\)-identities
11B68 Bernoulli and Euler numbers and polynomials

Keywords:
Catalan numbers

Software:
OEIS

Full Text: DOI arXiv

References:
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Edited by FIZ Karlsruhe, the European Mathematical Society and the Heidelberg Academy of Sciences and Humanities
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