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Let $X$ be a projective manifold equipped with an ample line bundle $A$. The existence of a constant scalar curvature Kähler metric ($cscK$) in class $c_1(A)$ is a central problem in Kähler geometry. The famous Yan-Tian-Donaldson conjecture states that $(M, A)$ admits a $cscK$ (in the class of $c_1(A)$) if and only if $(M, A)$ is $K$-polystable. The difficult part of the conjecture is the “if” part, which is largely open. Note that $K$-stability is not the only GIT stability notion related to the existence of $cscK$ metrics. Assuming that $H^0(M, T_M) = 0$, S. K. Donaldson [J. Differ. Geom. 59, No. 3, 479–522 (2001; Zbl 1052.32017)] proved that the existence of $cscK$ in $c_1(A)$ implies asymptotic Chow stability. Thanks to the work of T. Mabuchi [Invent. Math. 159, No. 2, 225–243 (2005; Zbl 1118.53047)], the assumption $H^0(M, T_M) = 0$ can be removed by introducing some hypothesis between $A$ and Aut $(M)$. These invariants are called higher Futaki invariants.

The article under review first proves that, like the original Futaki invariant, the higher Futaki invariants have an algebraic-geometric nature. Furthermore, the authors discuss some properties of the asymptotic Chow stability for two types of manifolds. The authors prove that, for a projective bundle $P(E)$ over a curve of genus $g \geq 2$, asymptotic Chow stability is equivalent to slope polystability. In the case of blowups, they give an explicit formula for the Chow weight and higher Futaki invariants in terms of the data on the base manifolds. Combining this with a result of C. Arezzo and F. Pacard [Ann. Math. (2) 170, No. 2, 685–738 (2009; Zbl 1202.53069)], they prove that, if $M$ is the blow up at four points of $P^2$ (all but one are aligned), then $M$ admits an asymptotically Chow unstable $cscK$ polarization.

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References:

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