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On the Tonelli’s partial regularity. (English) Zbl 1289.49041

The author studies regularity properties of minima of variational problems with one independent variable, i.e.

$$\min \left\{ \int_a^b L(t, u(t), u'(t)) \, dt \mid u \in W^{1,1}_0(a, b) \right\}.$$

The main result is that minima are regular in the sense of Tonelli, meaning that the derivative of $u$ is continuous as an extended real-valued function outside an exceptional set of measure zero. The novelty is in assuming that $L$ is invariant under a certain group of $C^1$ transformations. This allows relaxing the assumption about the dependency of $L$ on $u$ to just continuity; a typical assumption needed for this type of result is Lipschitz continuity.

Reviewer: Teemu Lukkari (Trondheim)

MSC:

49N60 Regularity of solutions in optimal control

Keywords:

one-dimensional variational problem; Tonelli’s partial regularity; symmetry; Noether’s theorem