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Homogenization and dispersion effects in the problem of propagation of waves generated by a localized source. (English. Russian original) Zbl 1293.35279

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Homogenization methods are effectively used in linear partial differential equations with rapidly oscillating coefficients and smooth coefficients at leading terms for the construction of their asymptotic solutions. The aim of this article is to modify the homogenization method when the leading term is also rapidly oscillating on the example of the Cauchy problem with rapidly decreasing initial data for the wave equation in the plane $\mathbb{R}^2_x$ with oscillating coefficients

$$u_{tt} = \left\langle \nabla, C^2\left(\frac{\theta(x)}{\varepsilon}, x\right)\nabla\right\rangle u,\ x = x_1, x_2;\ u(x,0) = V\left(\frac{x}{\mu}\right),\ u_t(x,0) = 0.$$

Here $\theta(x) = (\theta_1, \theta_2)$, $C(y,x)$ are smooth, $C^2(y,x) > \kappa_0 > 0$ is 2π-periodic on y with some additional assumptions, ε and $\mu = \varepsilon^\kappa$, $1 > \kappa \geq 0$ are small parameters characterizing the oscillation rate and the decay in initial data. Such problems arise in the study of the propagation of tsunami waves.

The asymptotic solution of such problems has dispersion effects that are studied here by means of a modified Maslov canonical operator. As a result, the constructed asymptotic solution expressed through the Airy functions allows one to investigate the effect of fast velocity oscillations on the solution fronts.

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MSC:

35Q53 KdV equations (Korteweg-de Vries equations)
35B27 Homogenization in context of PDEs; PDEs in media with periodic structure
35Q35 PDEs in connection with fluid mechanics
76B15 Water waves, gravity waves; dispersion and scattering, nonlinear interaction

Keywords:

wave equation; rapidly oscillating coefficients; decreasing initial data; asymptotic solution; homogenization method modification; geophysical applications

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