De Philippis, G.; Maggi, F.
Sharp stability inequalities for the Plateau problem. (English) Zbl 1293.49103
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Let $\mathcal{M}$ be the family of smooth, compact, orientable hypersurfaces $M \subset \mathbb{R}^{n+1}$ with smooth boundary $\mathrm{bd}(M)$. A hypersurface $M \in \mathcal{M}$ is said to be uniquely area minimizing in $\mathcal{M}$ if $\mathcal{H}^n(M') \geq \mathcal{H}^n(M)$ for all $M' \in \mathcal{M}$ with $\mathrm{bd}(M') = \mathrm{bd}(M)$, and $\mathcal{H}^n(M') = \mathcal{H}^n(M')$ if and only if $M = M'$, where $\mathcal{H}^n$ is the n-dimensional Hausdorff measure on $\mathbb{R}^{n+1}$. If $M, M' \in \mathcal{M}$, then there exists a Borel set $E \subset \mathbb{R}^{n+1}$ with finite Lebesgue measure $\mathcal{L}^{n+1}(E)$ bounded by $M \Delta M' = (M \setminus M') \cup (M' \setminus M)$. The problem is to find necessary and sufficient conditions for the existence of a positive constant κ such that, if $\mathrm{bd}(M') = \mathrm{bd}(M)$, then the global stability inequality $(*)\, \mathcal{H}^n(M') - \mathcal{H}^n(M) \geq \kappa \min\{\mathcal{L}^{n+1}(E)^2, \mathcal{L}^{n+1}(E)^{n/(n+1)}\}$ holds. For a smooth $\varphi : M \to \mathbb{R}$ vanishing on $\mathrm{bd}(M)$ let $\nabla^M \varphi$ denote the tangential gradient of φ with respect to M. For the second fundamental form II_M of M, let the first eigenvalue of the second variation of the area be strictly positive at M, that is, there exists $\lambda > 0$ such that $(**)\ \int_M |\nabla^M \varphi|^2 - |II_M|^2 \varphi^2 d\,\mathcal{H}^n \geq \lambda \int_M \varphi^2 d\,\mathcal{H}^n$.

In this paper, the authors show that if $M \in \mathcal{M}$ is uniquely area minimizing in $\mathcal{M}$, then the global stability inequality $(*)$ is equivalent to the positivity of the second variation $(**)$ of the area at M. They extend this result to sets uniquely mass minimizing as an integral n-current. Also, considering singular area minimizing hypersurfaces, the authors show quadratic stability inequalities with explicit constants for all the Lawson's cones, excluding six exceptional cases. Finally, they find explicit lower bounds for the first eigenvalues of the second variation of the area on these cones.

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