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Lenz, Simon Manfred

Impulsive hybrid discrete-continuous delay differential equations. (English) Zbl 1294.65073
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Summary: This thesis deals with impulsive hybrid discrete-continuous delay differential equations (IHDDEs). This new class of differential equations is highly challenging for two reasons. First, because of a dependency of the right-hand-side function on past states, with time delays that depend on the current state. Second, because both the right-hand-side function and the state itself are discontinuous at implicitly defined time points.

The theoretical results and numerical methods presented in this thesis are related to the following subject areas: First, solutions of initial value problems (IVPs) in IHDDEs. Second, derivatives of IVP solutions with respect to parameters ("sensitivities"). Third, estimation of parameters in IHDDE models from experimental data. Amongst others, this thesis thereby makes the following contributions:

– The theoretical basis of IHDDE-IVPs is established. This includes the definition of a solution concept, the existence of solutions, the uniqueness of solutions, and the differentiability of solutions with respect to parameters.

– A new approach for numerically solving IVPs in differential equations with time delays is introduced. A key aspect is the use of extrapolations beyond past discontinuities. Convergence of continuous Runge-Kutta methods realized in the framework of the new approach is shown, and numerical results are presented that demonstrate the benefit of using extrapolations on a practical example.

– A "first discretize, then differentiate" approach and a "first differentiate, then discretize" approach for forward sensitivity computation in IHDDEs are investigated. It is revealed that the presence of time delays destroys commutativity of differentiation and discretization in the case of continuous Runge-Kutta methods.

– An extension of the concept of internal numerical differentiation is proposed for differential equations with time delays. The use of the extended concept ensures that numerically computed sensitivities converge to the exact sensitivities, and that the convergence order is identical to the convergence order of the method that is used for solving the nominal IVP.

– The first practical forward and adjoint schemes are developed that realize Internal Numerical Differentiation for IHDDEs. Numerical investigations show that the developed schemes are drastically more efficient than classical methods for sensitivity computation.

– The new numerical methods for solving IVPs and for computing sensitivites are successfully applied to several challenging test cases, and the properties of the methods are analysed.

– Numerical methods are presented for solving nonlinear least-squares parameter estimation problems constrained by IHDDEs.

– A new epidemiological IHDDE model is developed. Therein, an impulse accounts for the arrival of an infected population. Further, the zeros of state-dependent switching functions characterize the time points at which new medical treatments become available.

– A delay differential equation model is presented for the crosstalk of the signaling pathways of two cytokines. In comparison to an ordinary differential equation model, a better fit to experimental data is obtained with a smaller number of differential states.

– A novel model is proposed to describe the voting behavior of the viewers of the TV singing competition "Unser Star für Baku" aired in 2012. Numerical results show that the use of a time delay is crucial for a qualitative correct description of the voting behavior. Furthermore, parameter estimation results yield a good quantitative agreeement with data from the TV show.

– The practical implementation of all developed methods in the new software packages Colsol-DDE and ParamEDE is described.

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MSC:

Cited in **1** Document

65L03	Numerical methods for functional-differential equations
34K28	Numerical approximation of solutions of functional-differential equations (MSC2010)
34K34	Hybrid systems of functional-differential equations
34K45	Functional-differential equations with impulses
65L06	Multistep, Runge-Kutta and extrapolation methods for ordinary differential equations
65L05	Numerical methods for initial value problems involving ordinary differential equations
65L20	Stability and convergence of numerical methods for ordinary differential equations
65D25	Numerical differentiation

Keywords:

hybrid discrete-continuous delay differential equations; initial value problem; convergence; Runge-Kutta method; numerical result; extrapolation; internal numerical differentiation; nonlinear least-squares parameter estimation

Software:

ParamEDE; DASPKADJOINT; DASPK 3.0; Colsol-DDE

Full Text: Link

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