A $k$-fold $x$-colouring of a graph is an assignment of (at least) $k$ distinct colours from the set \{1, 2, \ldots, x\} to each vertex such that any two adjacent vertices are assigned disjoint sets of colours. The smallest number $x$ such that a graph $G$ admits a $k$-fold $x$-colouring is the $k$th chromatic number of $G$, denoted by $\chi_k(G)$.

The paper deals with graphs that are called webs and antiwebs, where a web is a graph $W_{n, p}$ with vertex set \{v_0, v_1, \ldots, v_n\}, $p \geq 1$, $n \geq 2p$ and edge set \{v_i v_j : p \leq |i - j| \leq n - p\} and an antiweb is its complement. The authors determine exact values of the studied invariant for webs and antiwebs. The obtained results provide a generalization of the known results for odd cycles and imply necessary and sufficient conditions under which $\chi_k(G)$ attains its lower and upper bounds based on clique and integer and fractional chromatic numbers. Moreover, the authors introduce the concept of $\chi_k@$-critical graphs.

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MSC:

05C15 Coloring of graphs and hypergraphs

Keywords:

$k$-fold graph coloring; fractional chromatic number; clique number; stable set number; web; antiweb

Full Text: DOI arXiv