Let $R$ be a ring, not necessarily commutative, with unit. Let $Z(R)$ be the set of all zero-divisors of $R$. The total graph of $R$, denoted by $T(\Gamma(R))$ is the graph with all elements of $R$ as vertices where two distinct vertices $x, y \in R$ are adjacent iff $x + y \in Z(R)$. Let the regular graph of $R$, denoted by $\text{Reg}(\Gamma(R))$, be the induced subgraph of $T(\Gamma(R))$ on the regular elements of $R$. Let $Z(\Gamma(R))$ be the induced subgraph of $T(\Gamma(R))$ on $Z(R)$.

In this paper, the authors show that $\text{gr}(\text{Reg}(\Gamma(R)))$ and $\text{gr}(T(\Gamma(R))) \in \{3, 4, \infty\}$.

Also, the following results are proved.

Theorem. Let $R$ be a left Artinian ring and $\text{Reg}(\Gamma(R))$ be a tree. Then $R$ is isomorphic to one of the following rings, $Z_1, Z_4, Z_2[x]/(x^2), Z_2^2, Z_4 \times Z_2^2, Z_4 \times Z_2^2, Z_2[x]/(x^2) \times Z_2^2, \text{UT}_2\mathbb{Z}_2, \text{UT}_2\mathbb{Z}_2 \times Z_2^2$, where $\text{UT}_2(\mathbb{Z}_2)$ denotes the ring of $2 \times 2$ upper triangular matrices over $\mathbb{Z}_2$ and $r$ is a natural number.

Theorem. Let $R$ be a left Noetherian ring and $2 \notin Z(R)$. If $R$ is reduced, then $\chi(\text{Reg}(\Gamma(R))) = \omega(\text{Reg}(\Gamma(R))) = 2^r$, where $r$ is the number of minimal prime ideals of $R$.

Theorem. Let $R$ be a left Artinian ring. If $\text{Reg}(\Gamma(R))$ contains a vertex adjacent to all other vertices, then $\text{Reg}(\Gamma(R))$ is complete.

Theorem. Let $R$ be a semiprime left Noetherian ring. If $\text{Reg}(R)$ is finite, then $R$ is finite.

The authors also pose the conjecture: “Suppose that $R$ is a left Noetherian ring. If $\text{Reg}(R)$ is finite, then $R$ is finite”.

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MSC:
- 05C25 Graphs and abstract algebra (groups, rings, fields, etc.)
- 05C15 Coloring of graphs and hypergraphs
- 16U99 Conditions on elements

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- regular graph
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- girth
- chromatic number
- Noetherian ring
- Artinian ring

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