Special curves on a variety reflect the geometry of the variety itself. Along this direction, the notion of rationally connected varieties was first introduced by J. Kollár et al. [J. Algebr. Geom. 1, No. 3, 429–448 (1992; Zbl 0780.14026)] and F. Campana [Ann. Sci. Éc. Norm. Supér. (4) 25, No. 5, 539–545 (1992; Zbl 0783.14022)]. A variety $X$ is rationally connected if a general pair of points can be connected by a rational curve. It turns out that on a rationally connected variety, there is a rational curve passing through any finitely many points. After fixing an ample divisor $D$ on $X$, one can put restriction on the degree of the rational curves connecting a pair of points. The classification of varieties whose general pair of points can be connected by a conic is done by P. Ionescu and F. Russo [J. Reine Angew. Math. 644, 145–157 (2010; Zbl 1200.14078)]. The case of rationally cubic connected varieties is treated in [G. Occhetta and V. Paterno, J. Math. Soc. Japan 64, No. 3, 941–967 (2012; Zbl 1260.14064)] and [G. Occhetta and V. Paterno, Rev. Mat. Iberoam. 28, No. 3, 815–838 (2012; Zbl 1260.14063)].

The work under review is a continuation of [L. Pirio and J.-M. Trépreau, Bull. Soc. Math. Fr. 141, No. 1, 131–196 (2013; Zbl 1276.14079)]. Let $X = X(r + 1, n, \delta)$ be a projective variety of dimension $r + 1$ such that through $n \geq 2$ general points there passes an irreducible curve of degree $\delta \geq n - 1$ with respect to certain fixed Cartier divisor $D$. The authors obtain a bound for the dimension of the linear system $|D|$ in terms of the Castelnuovo-Harris bound function for geometric genus. This further gives a bound for the self-intersection number of $D$, which generalises a result of Fano. While the classification of all such $X$ seems out of reach, the authors obtain a list of smooth $X$ with $n = 3$ and $\delta = 3$. Many interesting examples are discussed in Section 4.

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