Pavlov, V. P.

Differential-geometric aspects of a nonholonomic Dirac mechanics: lessons of a model quadratic in velocities. (English. Russian original) Zbl 1298.81129

Theor. Math. Phys. 178, No. 3, 347-358 (2014); translation from Teor. Mat. Fiz. 178, No. 3, 403-415 (2014).

Summary: Faddeev and Vershik proposed the Hamiltonian and Lagrangian formulations of constrained mechanical systems that are invariant from the differential geometry standpoint. In both formulations, the description is based on a nondegenerate symplectic 2-form defined on a cotangent bundle T^*Q (in the Hamiltonian formulation) or on a tangent bundle TQ (in the Lagrangian formulation), and constraints are sets of functions in involution on these manifolds. We demonstrate that this technique does not allow "invariantization" of the Dirac procedure of constraint "proliferation". We show this in an example of a typical quantum field model in which the original Lagrange function is a quadratic form in velocities with a degenerate coefficient matrix. We postulate that the initial phase space is a manifold where all arguments of the action functional including the Lagrange multipliers are defined. The Lagrange multipliers can then be naturally interpreted physically as velocities (in the Hamiltonian formulation) or momenta (in the Lagrangian formulation) related to "nonphysical" degrees of freedom. A quasisymplectic 2-form invariantly defined on such a manifold is degenerate. We propose new differential-geometric structures that allow formulating the Dirac procedure invariantly.

MSC:

81S10	Geometry and quantization, symplectic methods
70H45	Constrained dynamics, Dirac's theory of constraints
70F20	Holonomic systems related to the dynamics of a system of particles
81T70	Quantization in field theory; cohomological methods

Keywords:

nonholonomic Dirac mechanics; constraint proliferation; differential geometry

Full Text: DOI

References:

[1] L. D. Faddeev, Theor. Math. Phys., 1, 1–13 (1969). · Zbl 1183.81090 · doi:10.1007/BF01028566

[2] V. I. Arnol'd, Mathematical Methods of Classical Mechanics [in Russian], Nauka, Moscow (1989); English transl. (Grad. Texts in Math., Vol. 60), Springer, New York (1989).

[3] A. M. Vershik and L. D. Faddeev, Sov. Phys. Dokl., 17, 34–36 (1972).

[4] A. M. Vershik and L. D. Faddeev, Selecta Math. Sov., 1, 339–350 (1981).

[5] A. M. Vershik, "Mathematics of nonholonomicity," in: The Thermodynamic Approach to Market (D. A. Leites, ed.) (Preprint No. 76), Max-Planck-Institut für Mathematik, Leipzig (2006), pp. 137–154; arXiv:0803.3432v1 [physics.soc-ph] (2008).

[6] C. Godbillon, Géométrie différentielle et mécanique analytique, Hermann, Paris (1969).

[7] P. A. M. Dirac, Proc. Roy. Soc. London A, 246, 326–332 (1958). · Zbl 0080.41402 · doi:10.1098/rspa.1958.0141

[8] V. P. Pavlov and A. O. Starinetz, Theor. Math. Phys., 105, 1539–1545 (1995). · Zbl 0925.70156 · doi:10.1007/BF02070875

[9] V. P. Pavlov, Proc. Steklov Inst. Math., 228, 135–144 (2000).

[10] V. P. Pavlov, Nonholonomic Dirac Mechanics and Differential Geometry [in Russian], MIAN Publ., Moscow (2013).