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**Bernstein inequality in $L^\alpha$ norms.** (English) Zbl 1299.41022


An extended version of Bernstein’s inequality in $L^\alpha$ norms is given as follows. Let $K \subset \mathbb{R}$ be a compact set consisting of finitely many disjoint closed intervals, none of them consisting of a single point. Let denote by $\nu_K$ its equilibrium measure and by $\omega_K(t)$ its density, $\omega_K(t) = d\nu_K(t)/dt$. Let $1 \leq \alpha < \infty$. The main result assures the following inequality

$$
\int_K \frac{|P_n(t)|}{n \pi \omega_K(t)} \, d\nu_K(t) \leq (1 + o(1)) \int_K |P_n(t)|^\alpha d\omega_K(t),
$$

where $P_n$ is an arbitrary polynomial of degree $n$ and $o(1)$ denotes an error term that tends to 0 as $n \to \infty$ and is independent of $P_n$. Moreover, the constant 1 in the right hand side of this inequality is optimal.

The proof is very technical. It contains many steps and it is carefully written to details. Among the results used in the proof we mention the Zygmund inequality of Bernstein type, Totik’s results on lemniscates of polynomials and a Nikolskii type inequality.

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**MSC:**

- 41A17 Inequalities in approximation (Bernstein, Jackson, Nikol’skiĭ-type inequalities)
- 26D05 Inequalities for trigonometric functions and polynomials
- 30C85 Capacity and harmonic measure in the complex plane

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polynomial inequalities; Bernstein inequality; potential theory; equilibrium measure; lemniscate of polynomial