For natural numbers \( n \) and \( k \), the rainbow Ramsey theorem (RRT\(^n_k\)) states that if \( f : [\mathbb{N}]^n \to \mathbb{N} \) satisfies \( |f^{-1}(c)| \leq k \) for each \( c \), then there is an infinite set \( R \) such that \( f \) is injective on \([R]^n\). \( R \) is called a rainbow for \( f \). The central results of this paper analyze the strength of RRT\(^3_2\) in the framework of reverse mathematics, showing that RCA\(_0\) + RRT\(^3_2\) implies neither WKL\(_0\) nor RRT\(^4_2\). Computability-theoretic results on cohesive sets are used in the proofs. The results here sharpen the main result of the author’s earlier related paper [J. Symb. Log. 78, No. 3, 824–836 (2013; Zbl 1300.03013)].

Reviewer: Jeffry L. Hirst (Boone)

MSC:

03B30  Foundations of classical theories (including reverse mathematics)
03F35  Second- and higher-order arithmetic and fragments
03D55  Hierarchies of computability and definability
03D80  Applications of computability and recursion theory
05D10  Ramsey theory

Keywords:

reverse mathematics; Ramsey’s theorem; rainbow Ramsey theorem; cohesive set; weak König’s lemma; WKL

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References:

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[8] Liu, Jiayi, $\forall \operatorname{operatorname{RT}}_2^2 \forall \operatorname{operatorname{WKL}}_0$ does not imply $\forall \operatorname{operatorname{RT}}_2^2$, J. Symbolic Logic, 77, 2, 609-620, (2012) · Zbl 1245.03005

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