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Long-time asymptotics of the periodic Toda lattice under short-range perturbations. (English) Zbl 1301.37055

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Summary: We compute the long-time asymptotics of periodic (and slightly more generally of algebro-geometric finite-gap) solutions of the doubly infinite Toda lattice under a short-range perturbation. In particular, we prove that the perturbed lattice asymptotically approaches a modulated lattice. More precisely, let g be the genus of the hyperelliptic curve associated with the unperturbed solution. We show that, apart from the phenomenon of solitons travelling in a quasi-periodic background, the n/t-pane contains $g+2$ areas where the perturbed solution is close to a finite-gap solution on the same isospectral torus. In between there are $g+1$ regions where the perturbed solution is asymptotically close to a modulated lattice which undergoes a continuous phase transition (in the Jacobian variety) and which interpolates between these isospectral solutions. In the special case of the free lattice ($g = 0$), the isospectral torus consists of just one point and we recover the known result. Both the solutions in the isospectral torus and the phase transition are explicitly characterized in terms of abelian integrals on the underlying hyperelliptic curve. Our method relies on the equivalence of the inverse spectral problem to a vector Riemann-Hilbert problem defined on the hyperelliptic curve and generalizes the so-called nonlinear stationary phase/steepest descent method for Riemann-Hilbert problem deformations to Riemann surfaces.

MSC:

37K40	Soliton theory, asymptotic behavior of solutions of infinite-dimensional Hamiltonian systems
37K10	Completely integrable infinite-dimensional Hamiltonian and Lagrangian systems, integration methods, integrability tests, integrable hierarchies (KdV, KP, Toda, etc.)
37K20	Relations of infinite-dimensional Hamiltonian and Lagrangian dynamical systems with algebraic geometry, complex analysis, and special functions
37K60	Lattice dynamics; integrable lattice equations
35Q15	Riemann-Hilbert problems in context of PDEs
14H52	Elliptic curves
14H70	Relationships between algebraic curves and integrable systems

Keywords:

long-time asymptotics; periodic Toda lattice; short-range perturbation; soliton

Full Text: DOI arXiv

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