Passeggi, Alejandro  
Rational polygons as rotation sets of generic homeomorphisms of the two torus. (English)  
Zbl 1301.54053  

Rotation theory in the two-dimensional torus can be seen as the first attempt to generalize Poincaré’s combinatorial theory on the dynamics of orientation-preserving circle homeomorphisms. In the present paper the author proves the existence of an open and dense set \( D \subset \text{Homeo}_0(T^2) \) (where \( \text{Homeo}_0(T^2) \) denotes the set of toral homeomorphisms homotopic to the identity) such that the rotation set of any element in \( D \) is a rational polygon. This result can be seen as a generalization of the corresponding result in the circle theory, which asserts that for an open and dense set in \( \text{Homeo}_0(S^1) \) the rotation number of any element belonging to this set is rational. The author also extends this result to the set of Axiom A diffeomorphisms in \( \text{Homeo}_0(T^2) \). Recall that a \( C^1 \)-diffeomorphism \( f \) acting on a compact Riemannian manifold satisfies Axiom A if the nonwandering set of \( f \) is hyperbolic and the set of periodic points of \( f \) is dense in the nonwandering set. Examples include the Anosov diffeomorphisms and Smale’s horseshoe map. Furthermore, the author observes the existence of minimal sets whose rotation set is a non-trivial segment, for an open set in \( \text{Homeo}_0(T^2) \).

Reviewer: Antonios Manoussos (Bielefeld)

MSC:
54H20  Topological dynamics (MSC2010)  
37E45  Rotation numbers and vectors  
37E30  Dynamical systems involving homeomorphisms and diffeomorphisms of planes and surfaces  
37D20  Uniformly hyperbolic systems (expanding, Anosov, Axiom A, etc.)

Full Text: DOI arXiv