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Adjoint groups of $p$-nil rings and $p$-group automorphisms. (English) [Zbl 1302.20027]

Let $N$ be an Abelian normal subgroup of the group $G$. H. Laue [J. Algebra 96, 532-547 (1985; Zbl 0573.20028)] proved that there is an isomorphism between the monoid $\text{End}_N(G)$ of the endomorphisms of $G$ that leave $N$ invariant, and induce the identity on $G/N$, and the adjoint monoid of the ring $\text{Der}(G, N)$. The latter set consists of the derivations of $G$ into $N$, and becomes a ring under addition and composition; the adjoint monoid has as operation the circle $x \circ y = x + y + xy$.

Laue’s result has proved very useful in investigating automorphisms of groups, in particular finite $p$-groups. The paper under review follows up Laue’s ideas. The authors define a ring $R$ to be left (right) $p$-nil, where $p$ is an odd prime, if every element of additive order $p$ is a left (right) annihilator. They show among others that the power structure of the adjoint group of a left or right $p$-nil ring is very close to that of the additive group of the ring. They then apply their results to the study of automorphism groups of finite $p$-groups. A typical result is that if $G$ is a finite $p$-group in which the center is contained in the Frattini subgroup, then the group of central automorphisms, that is, those automorphisms that induce the identity on the central quotient $G/Z(G)$, has nilpotence class at most $\min\{r, s\}$, where $p^r$ is the exponent of the Abelianization $G/G'$, and $p^s$ is the exponent of the center.

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20D45 Automorphisms of abstract finite groups
16N20 Jacobson radical, quasimultiplication
20D15 Finite nilpotent groups, $p$-groups

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nilpotent rings; adjoint groups; finite $p$-groups; central automorphisms; right $p$-nil rings; automorphism groups

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