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Keller-box method and its application. (English) [Zbl 1305.76002](#)

De Gruyter Studies in Mathematical Physics 8. Berlin: de Gruyter; Beijing: Higher Education Press (ISBN 978-3-11-027137-9/hbk; 978-3-11-027178-2/ebook). xi, 401 p. (2014).

The history of the Keller-box method started in the early 1970's, when *H. B. Keller* published the paper ["A new difference scheme for parabolic problems", in: Numerical Solution partial diff. Equations II, Proc. 2nd Sympos. numerical Solution partial diff. Equations, SYNSPADE 1970, Univ. Maryland, 327–350 (1971; [Zbl 0243.65060](#))]. Around that time, there appeared many important problems which combined physics, numerical mathematics, and, to some extent, computer science employed to simulate fluid flow and heat transfer phenomena. Nowadays, Keller-box methodologies are routinely employed in the fields of fluid mechanics, heat transfer theory, aviation, meteorology, oceanography, astrophysics, architecture, etc. This powerful method is thus becoming an increasingly important design tool in engineering and also a substantial research tool in the physical sciences.

The present book provides university students and researchers in applied sciences with a solid foundation for understanding the numerical methods employed in solving today's many practical problems, and familiarizes them with modern numerical code by hands-on experience. The reader may be advised to additionally read the older textbook of *T. Cebeci* and *P. Bradshaw* [Physical and computational aspects of convective heat transfer. New York etc.: Springer-Verlag (1984; [Zbl 0545.76090](#))] for comparison.

The book under review consists of a preface, the contents, seven chapters, a subject index and an author index. The description of these chapters is, in short, as follows:

Chapter 0 "Introduction" shows that by using the Keller-box or, in short, the box method, numerous nonlinear differential equations can be studied in great detail.

Chapter 1 "Basics of the finite difference approximations" presents the essentials of finite difference approximations for solving linear/nonlinear differential equations. It treats finite difference approximations; initial value problem for ODEs; some basic numerical methods (one-step errors, Taylor series methods, Runge-Kutta methods, linear multi-step methods, along with 8 very interesting examples); some basic PDEs; numerical solutions to partial differential equations, and a list of 10 references. The authors also present here the basic components of a numerical method (mathematical model, discretization, finite approximation, convergence), and the properties of numerical methods (consistence, stability, convergence, etc.).

Chapter 2 "Principles of the implicit Keller-box method" refers to the basic concepts of finite difference methods for solving a system of linear homogeneous first-order differential equations (linear second-order differential equations, derivative boundary conditions, solutions of tridiagonal systems, iteration methods, the Newton-Raphson method, convergence of difference schemes, stability of finite difference schemes); boundary value problems for ordinary differential equations (second-order equations, fourth-order equations, shooting method, Keller-box method) along with many interesting examples.

Chapters 3 "Stability and convergence of the implicit Keller-box method" is an important chapter which systematically describes the properties of the implicit Keller-box method (convergence of implicit difference methods for parabolic functional differential equations; rate of convergence of finite difference schemes on uniform/non-uniform grids; stability and convergence of the Crank-Nicolson method for fractional advection dispersion equations; radial flow problem).

Chapter 4 "Application of the Keller-box method to boundary layer problems" discusses the application of the Keller-box method to several viscous incompressible Newtonian and non-Newtonian fluid flow and heat transfer problems over an infinite flat plate: flows of a power-law fluid over a stretching sheet (introduction, formulation of the problem, numerical solution method, results and discussion); hydromagnetic flows of a power-law fluid over a stretching sheet (introduction, flow analysis, numerical solution methods, results and discussion); MHD power-law fluid flow and heat transfer over a non-isothermal stretching sheet (introduction, governing equations and similarity analysis, heat transfer, numerical procedure, results and discussion); MHD flow and heat transfer of a Maxwell fluid over a non-isothermal stretching sheet (introduction, mathematical formulation, heat transfer analysis, numerical procedure, results and

discussion, conclusions); MHD boundary layer flow of a micropolar fluid past a wedge with constant wall heat flux (introduction, flow analysis, flat plate problem, results, discussion and conclusions). The chapter ends with a list of 109 references.

Chapter 5 “Application of the Keller-box method to fluid flow and heat transfer problems” describes the use of the Keller-box method to solve coupled nonlinear boundary value problems, such as: hydromagnetic flow and heat transfer adjacent to a stretching vertical sheet; convection flow and heat transfer of a Maxwell fluid over a non-isothermal surface; the effects of variable fluid properties on the hydromagnetic flow and heat transfer over a nonlinearly stretching sheet; the effects of linear/nonlinear convection on the non-Darcian flow and heat transfer along a permeable vertical surface; and unsteady flow and heat transfer in a thin film of Ostwald-de Waele liquid over a stretching surface. The chapter ends with a list of 126 references.

Chapter 6 “Application of the Keller-box method to more advanced problems” deals with coupled nonlinear boundary value problems of one or more independent variables: heat transfer phenomena in a moving nanofluid over a horizontal surface; hydromagnetic fluid flow and heat transfer at a stretching sheet with fluid-particle suspension and variable properties; radiation effects on mixed convection over a wedge embedded in a porous medium filled with a nanofluid; MHD mixed convection flow over a permeable non-isothermal wedge; mixed convection boundary layer flow about a solid sphere with Newtonian heating; and flow and heat transfer of a viscoelastic fluid over a flat plate with a magnetic field and a pressure gradient. A list of 156 references is included in this chapter. The book ends with a subject index and an author index.

In the reviewer’s opinion, this book provides a fundamental and comprehensive presentation of mathematical principles of the Keller-box method and its applications to numerical solutions of problems of practical interest. The references included in this book are very important, especially for young researchers who wish to deal with numerical methods for fluid dynamics and heat transfer. Moreover, Chapter 6 provides a solid background for future research in the newer field of nanofluids. The book is very well written and readable. Results of the numerical solutions of the considered problems are given graphically and in tabular form. The book will be of interest to a wide range of specialists working in different areas of fluid mechanics and heat transfer, such as graduate, MSc and PhD students, engineers, physicists, chemical engineers, and also to researchers interested in the mathematical theory of fluid mechanics and in connected topics.

Reviewer: [Ioan Pop \(Cluj-Napoca\)](#)

MSC:

- [76-02](#) Research exposition (monographs, survey articles) pertaining to fluid mechanics Cited in **3** Documents
- [76M20](#) Finite difference methods applied to problems in fluid mechanics
- [76R10](#) Free convection
- [76D10](#) Boundary-layer theory, separation and reattachment, higher-order effects
- [76W05](#) Magnetohydrodynamics and electrohydrodynamics
- [76A05](#) Non-Newtonian fluids
- [80A20](#) Heat and mass transfer, heat flow (MSC2010)

Keywords:

[finite difference method](#); [stability](#); [convergence](#); [boundary layer](#); [non-Newtonian fluid](#); [heat transfer](#); [MHD mixed convection](#)

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