The main result of this paper is Theorem 1.1: For every $\varepsilon \in (0, 1)$, there exists $C_\varepsilon \in (0, \infty)$ such that, if $(X, d)$ is a compact metric space and $\mu$ a Borel probability measure on $X$, then there exists a compact subset $S$ of $X$ which embeds into an ultrametric space with distortion $O\left(\frac{1}{\varepsilon}\right)$ and there exists a Borel probability measure $\nu$ on $S$ such that $\nu(B_d(x, r)) \leq (\mu(B_d(x, C_\varepsilon r)))^{1-\varepsilon}$, for all $x \in X$, $r \geq 0$. The two main ideas are to study the link between Theorem 1.1 and nonlinear Dvoretzky theorems and then to look at what happens when replacing $\mu$ by a finite number of Borel probability measures on $X$.

First, the link between this result and nonlinear Dvoretzky theorems is investigated. In a first step, the authors give examples to prove that the distortions to embed a subspace of an $n$-points metric space into a Hilbert space or an ultrametric space obtained in [M. Mendel and A. Naor, J. Eur. Math. Soc. (JEMS) 9, No. 2, 253–275 (2007; Zbl 1122.68043)] and [M. Mendel and A. Naor, Invent. Math. 192, No. 1, 1–54 (2013; Zbl 1272.30082)] are sharp. On the other hand, Theorem 1.1 connects nonlinear Dvoretzky problems considered in [Zbl 1272.30082] with a nonlinear Dvoretzky theorem of Talagrand and thus to Talagrand’s majorizing measure theorem; in order to prove this, the authors introduce the constants $\delta_2$ and $\gamma_2$ and compare them when computed over metric spaces, subsets and ultrametric spaces.

Theorem 1.4 asserts that, if $U_1$ and $U_2$ are two subsets of a metric space such that $U_i$ embeds into an ultrametric space with distortion $D_i$, then $U_1 \cup U_2$ embeds into an ultrametric space with distortion at most $(D_1 + 2)(D_2 + 2) - 2$. The sharpness of this bound is discussed. As a consequence, one can obtain a result of the type of Theorem 1.1 for a finite number of Borel probability measures $\mu_1, \ldots, \mu_k$.

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46B85 Embeddings of discrete metric spaces into Banach spaces; applications in topology and computer science
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