Albers, Peter; Frauenfelder, Urs; Solomon, Jake P.
A $\Gamma$-structure on Lagrangian Grassmannians. (English) Zbl 1308.53074

The (unoriented) Lagrangian Grassmannian $L$ is the space of all Lagrangian subspaces of $\mathbb{R}^{2n}$. It can be considered as the homogeneous space $L = U(n)/O(n)$.

A closed, connected, orientable manifold $M$ is said to have the structure of a $\Gamma$-manifold if there exists a map $\Theta : M \times M \to M$ such that the maps $x \to \Theta(x, y_0)$ and $y \to \Theta(x_0, y)$ have non-zero mapping degree for all pairs $(x_0, y_0) \in M \times M$.

The main result of this paper states that if $n$ is odd, then $(L, \Theta)$ together with some naturally defined smooth map $\Theta : L \times L \to L$ is a $\Gamma$-manifold. As a corollary it is reproved that for $n$ odd the rational cohomology ring of $L$ is an exterior algebra with generators of odd degree.

Reviewer: V. Gorbatsevich (Moskva)

MSC:
53C30 Differential geometry of homogeneous manifolds
53D12 Lagrangian submanifolds; Maslov index

Keywords:
Lagrangian Grassmanian; Gamma manifold; mapping degree

Full Text: DOI arXiv