Gateva-Ivanova, Tatiana; Fløystad, Gunnar
Monomial algebras defined by Lyndon words. (English) [Zbl 1310.16018]

D. J. Anick studied a class of monomial algebras with finite global dimension [Trans. Am. Math. Soc. 291, 291-310 (1985; Zbl 0575.16002); ibid. 296, 641-659 (1986; Zbl 0598.16028)]. This paper is a further study of graded associative algebras for which the set of obstructions consists of Lyndon words.

Let \( X = \{x_1, \ldots, x_g\} \) be a finite alphabet and \( K \) a field. The authors study monomial algebras \( A = K\langle X \rangle / (W) \), where \( W \) is an antichain of Lyndon words in \( X \) of arbitrary cardinality. The authors establish a Poincaré-Birkhoff-Witt type basis of \( A \) in terms of its Lyndon atoms \( N \); in general, \( N \) may be infinite.

The authors prove that if \( A \) has polynomial growth of degree \( d \) then \( A \) has global dimension \( d \) and is standard finitely presented, with \( d - 1 \leq |W| \leq d(d - 1)/2 \). Furthermore, \( A \) has polynomial growth iff the set of Lyndon atoms \( N \) is finite. In this case \( A \) has a \( K \)-basis \( \mathcal{N} = \{l_1^{\alpha_1}l_2^{\alpha_2} \cdots l_d^{\alpha_d} | \alpha_i \geq 0, 1 \leq i \leq d\} \). The authors also construct an extremal class of monomial algebras, called the Fibonacci-Lyndon algebras, \( F_n \), with global dimension \( n \) and of polynomial growth and uniquely determined up to isomorphism. The authors prove that the algebra \( F_6 \) of global dimension 6 cannot be deformed, keeping the multigrading, to an Artin-Schelter regular algebra.

Reviewer: Victor Petrogradsky (Brasilia)

MSC:
16P90 Growth rate, Gelfand-Kirillov dimension
16S15 Finite generation, finite presentability, normal forms (diamond lemma, term-rewriting)
68R15 Combinatorics on words
16W50 Graded rings and modules (associative rings and algebras)
16E10 Homological dimension in associative algebras
16Z05 Computational aspects of associative rings (general theory)

Keywords:
Lyndon words; monomial algebras; algebras of polynomial growth; global dimension; Artin-Schelter regular algebras; graded algebras; Poincaré-Birkhoff-Witt bases

Full Text: DOI arXiv

References:


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