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Product equivalence of quasihomogeneous Toeplitz operators on the harmonic Bergman space. (English) Zbl 1310.47041

Stud. Math. 219, No. 2, 163-175 (2013).

Let dA denote the Lebesgue area measure on the unit disk D, normalized so that the measure of D equals 1, and let $L^2(D,dA)$ be the Hilbert space of Lebesgue square integrable functions on D. The harmonic Bergman space L^2_h is the closed subspace of $L^2(D,dA)$ consisting of the harmonic functions on D. The orthogonal projection from $L^2(D,dA)$ onto L^2_h is denoted by Q. Given $z \in D$, let $K_z(w) = 1/(1-w\bar{z})^2$ be the well-known reproducing kernel for the analytic Bergman space L^2_a consisting of all L^2-analytic functions on D. The well-known Bergman projection P is then the integral operator $Pf(z) = \int_D f(w)\overline{K_z(w)}\,dA(w)$ for $f \in L^2(D,dA)$. Thus, Q can be represented by $Qf = Pf + \overline{P\bar{f}} - Pf(0)$. For $u \in L^1(D,dA)$, the Toeplitz operator T_u with symbol u is the operator on L^2_h defined by $T_u f = Q(uf)$ for $f \in L^2_h$. This operator is always densely defined on the polynomials and not bounded in general. The authors are interested in the case where it is bounded in the L^2_h norm, and u is a T-function. Then T_u has the continuous extension. A function f is said to be quasihomogeneous of degree $k \in \mathbb{Z}$ if $f(re^{i\theta}) = e^{ik\theta}\varphi(r)$, where φ is a radial function. Let f_1 and f_2 be two quasihomogeneous T-functions on D. In this case, the authors prove that, if there exists a T-function f such that $T_{f_1}T_{f_2} = T_f$, then $T_{f_2}T_{f_1} = T_f$ and $T_{f_1}T_{f_2} = T_{f_2}T_{f_1}$.

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MSC:

47B35 Toeplitz operators, Hankel operators, Wiener-Hopf operators

Keywords:

Toeplitz operators; harmonic Bergman space; quasihomogeneous symbols

Full Text: DOI