Restrictions of Brownian motion. (Restrictions du mouvement brownien.)

Let \( \{B(t) : 0 \leq t \leq 1\} \) be a linear Brownian motion and let \( \dim \) denote the Hausdorff dimension. Let \( \alpha > \frac{1}{2} \) and \( 1 \leq \beta \leq 2 \). We prove that, almost surely, there exists no set \( A \subset [0,1] \) such that \( \dim A > \frac{1}{2} \) and \( B : A \to \mathbb{R} \) is \( \alpha \)-Hölder continuous. The proof is an application of Kaufman’s dimension doubling theorem. As a corollary of the above theorem, we show that, almost surely, there exists no set \( A \subset [0,1] \) such that \( \dim A > \frac{\beta}{2} \) and \( B : A \to \mathbb{R} \) has finite \( \beta \)-variation. The zero set of \( B \) and a deterministic construction witness that the above theorems give the optimal dimensions.

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Brownian motion; Hausdorff dimension; Hölder continuity

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References:

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