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Summary: We study the question of whether, for a given class of finite graphs, one can define, for each graph of the class, a linear ordering in monadic second-order logic, possibly with the help of monadic parameters. We consider two variants of monadic second-order logic: one where we can only quantify over sets of vertices and one where we can also quantify over sets of edges. For several special cases, we present combinatorial characterisations of when such a linear ordering is definable. In some cases, for instance for graph classes that omit a fixed graph as a minor, the presented conditions are necessary and sufficient; in other cases, they are only necessary. Other graph classes we consider include complete bipartite graphs, split graphs, chordal graphs, and cographs. We prove that orderability is decidable for the so called HR-equational classes of graphs, which are described by equation systems and generalize the context-free languages.

MSC:
03B25 Decidability of theories and sets of sentences
03B15 Higher-order logic; type theory (MSC2010)
03D05 Automata and formal grammars in connection with logical questions
05C75 Structural characterization of families of graphs
68Q45 Formal languages and automata

Keywords:
monadic second-order logic; definability; linear orders

Full Text: DOI arXiv