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Uniqueness of the de Sitter spacetime among static vacua with positive cosmological constant. (English) [Zbl 1318.53054](#)

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Summary: We prove that, among all $(n + 1)$ -dimensional spin static vacua with positive cosmological constant, the de Sitter spacetime is characterized by the fact that its spatial Killing horizons have minimal modes for the Dirac operator. As a consequence, the de Sitter spacetime is the only vacuum of this type for which the induced metric tensor on some of its Killing horizons is at least equal to that of a round $(n - 1)$ -sphere. This extends uniqueness theorems shown in Boucher et al. (Phys Rev D 30:2447, 1984). Chruściel [2]. to more general horizon metrics and to the non-single horizon case.

MSC:

53C40 Global submanifolds

53C80 Applications of global differential geometry to the sciences

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Keywords:

static vacuum; de Sitter space-time; Killing horizon; Dirac operator; differential geometry; global analysis

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