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Stateless one-way multi-head finite automata with pebbles. (English) Zbl 1318.68109

The paper under review investigates stateless (i.e., single-state) deterministic one-way multi-head finite automata with pebbles. Let us attempt a short description of the model based on the deterministic finite automaton. Such an automaton has a finite-state control that controls a reading head that scans the input letter by letter. The head can only stay at the current letter or move right to the next letter, which is also expressed by the “one-way” attribute. Each transition can also change the state of the finite control. The move of the head and the state change are uniquely determined (“deterministic”) by the current letter read from the input and the current state of the finite control. The input is accepted by the automaton if after processing the whole input the state is final. First, the multi-head extension adds several independent reading heads, all operating on the same input. These heads can move independently and each transition can now specify a move for each head. Similarly, the next moves and the next state can now depend on the letters read by all heads. Second, the model considered here has pebbles. A pebble is like a marker that can be dropped by each head onto a letter. Each head can sense dropped pebbles in addition to the letter. In fact, the model can utilize several distinguishable pebbles. Finally, the stateless restriction essentially removes the finite control. The next moves of the heads including their dropped pebbles depend only on the letters and pebbles sensed by all heads.

The expressive power of this model depending on the number of heads and the number of pebbles is investigated. It is first demonstrated that states can be traded for heads and/or pebbles to obtain a stateless device. For example, the $m$ original states can be coded (in binary) using $O(\log m)$ additional pebbles as long as the device has at least 2 heads. With a single head, this approach fails because pebbles do not increase the expressive power of devices with a single head. Similarly, the heads can also be used to code the state, but this requires an additional pebble and the construction is slightly more complicated in this case. Without the additional pebble, it is demonstrated to be impossible in general. Next, the authors establish two infinite hierarchies: one based on the number of heads and another one based on the number of pebbles. This is first demonstrated for the heads using an explicit language family as evidence for the strictness. The hierarchy for the number of pebbles is slightly easier to establish and essentially uses a length restriction of the recognized languages. The natural question whether heads can be traded for pebbles and vice versa is also discussed. It turns out that neither can be traded for the other in general. Last, but not least, decidability questions are investigated. Since the authors demonstrate how to encode successful computations of one-way cellular automata using a stateless device with just 2 heads and 1 pebble, most of the interesting decidability problems (emptiness, inclusion, finiteness, etc.) are undecidable.

The paper is rather technical, but should still be understandable to anyone with a solid background in automata theory. Examples are provided whenever helpful and proofs are generally presented at a high level to explain the approach. Details are provided very often, but the technical proofs are often omitted. Overall, a nice reading that requires little detailed background.

Reviewer: Andreas Maletti (Stuttgart)

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References: