Freed, Alan David

Hypoelastic soft tissues. I: Theory. (English) Zbl 1320.74021

Acta Mech. 213, No. 1-2, 189-204 (2010).

Summary: Refinements are made to an existing hypoelastic theory developed by *A. D. Freed* ["Anisotropy in hypoelastic soft-tissue mechanics. I: Theory, II: Simple extensional experiments. J. Mech. Mater. Struct. 3, No. 5, 911–928 (2008), doi:10.2140/jomms.2008.3.911, ibid. 4, No. 6, 1005–1025 (2009), doi:10.2140/jomms.2009.4.1005] for the purpose of modeling the passive response of soft, fibrous, biological tissues. Oldroyd's operators [*J. G. Oldroyd*, Proc. R. Soc. A 200, 523–541 (1950; Zbl 1157.76305)] for convected differentiation and integration, which he derived from the tensor transformation law, are re-derived here from an integral equation defined in the polar configuration. Fields that obey these convected polar operators are said to be viable tensor fields, from which a new definition for strain and its rate are obtained and applied to a hypoelastic theory for tissue. Anisotropy is addressed through a material tensor, from which viable tensor fields describing fiber strain and strain rate are constructed. Material anisotropy and material constitution are handled separately for maximum flexibility. Isochoric hypoelastic models for isotropic, anisotropic, and fiber/matrix composite materials are derived. A material function is introduced to address special attributes that biological fibers impart on tissue behavior, four of which are proposed that represent various ways through which the fiber constituents might be described. Application to in-plane biaxial deformation is the focus of part II of this paper [Acta Mech. 213, No. 1–2, 205–222 (2010; Zbl 1320.74020)].

MSC:

74B20 Nonlinear elasticity
74L15 Biomechanical solid mechanics
74C99 Plastic materials, materials of stress-rate and internal-variable type
74E10 Anisotropy in solid mechanics

Full Text: DOI

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