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Solving the biharmonic Dirichlet problem on domains with corners. (English) Zbl 1325.74090
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Summary: The biharmonic Dirichlet boundary value problem on a bounded domain is the focus of the present paper. By Riesz' representation theorem the existence and uniqueness of a weak solution is quite direct. The problem that we are interested in appears when one is looking for constructive approximations of a solution. Numerical methods using for example finite elements, prefer systems of second equations to fourth order problems. *P. G. Ciarlet* and *P. A. Raviart* [in: Proc. Symp. Math. Aspects Finite Elem. Partial Diff. Equations 33, 125–145 (1974; [Zbl 0337.65058](#))] and *P. Monk* in [SIAM J. Numer. Anal. 24, 737–749 (1987; [Zbl 0632.65112](#))] consider approaches through second order problems assuming that the domain is smooth. We discuss what happens when the domain has corners. Moreover, we suggest a setting, which is in some sense between Ciarlet-Raviart and Monk, that inherits the benefits of both settings and that give the weak solution through a system type approach.

MSC:

[74K20](#) Plates
[35Q74](#) PDEs in connection with mechanics of deformable solids
[35D30](#) Weak solutions to PDEs

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