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**Structure of noncommutative solitons: existence and spectral theory.** (English)

Zbl 1327.35328

Lett. Math. Phys. 105, No. 10, 1377-1398 (2015).

Summary: We consider the Schrödinger equation with a Hamiltonian given by a second-order difference operator with nonconstant growing coefficients, on the half one-dimensional lattice. This operator appeared first naturally in the construction and dynamics of noncommutative solitons in the context of noncommutative field theory. We construct a ground state soliton for this equation and analyze its properties. In particular, we arrive at  $\ell^\infty$  and  $\ell^1$  estimates as well as a quasi-exponential spatial decay rate.

**MSC:**

**35Q40** PDEs in connection with quantum mechanics  
**35Q55** NLS equations (nonlinear Schrödinger equations)  
**39A05** General theory of difference equations

Cited in 4 Documents

**Keywords:**

noncommutative soliton; spectral theory; NLS; DNLS

**Full Text:** DOI arXiv

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