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Extension of derivations, and Connes-amenability of the enveloping dual Banach algebra.
(English) [Zbl 1328.47041]

Summary: If $D : A \to X$ is a derivation from a Banach algebra to a contractive, Banach $A$-bimodule, then one can equip $X^{**}$ with an $A^{**}$-bimodule structure, such that the second transpose $D^{**} : A^{**} \to X^{**}$ is again a derivation. We prove an analogous extension result, where $A^{**}$ is replaced by $F(A)$, the enveloping dual Banach algebra of $A$, and $X^{**}$ by an appropriate kind of universal, enveloping, normal dual bimodule of $X$.

Using this, we obtain some new characterizations of Connes-amenability of $F(A)$. In particular we show that $F(A)$ is Connes-amenable if and only if $A$ admits a so-called WAP-virtual diagonal. We show that when $A = L^1(G)$, existence of a WAP-virtual diagonal is equivalent to the existence of a virtual diagonal in the usual sense. Our approach does not involve invariant means for $G$.

MSC:
47B48 Linear operators on Banach algebras
47B47 Commutators, derivations, elementary operators, etc.
46H25 Normed modules and Banach modules, topological modules (if not placed in 13-XX or 16-XX)

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