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Bayesian Poisson log-bilinear models for mortality projections with multiple populations.

(English) Zbl 1329.91111
Eur. Actuar. J. 5, No. 2, 245-281 (2015).

Summary: Life insurers, pension funds, health care providers and social security institutions face increasing expenses due to continuing improvements of mortality rates. The actuarial and demographic literature has introduced a myriad of (deterministic and stochastic) models to forecast mortality rates of single populations. This paper presents a Bayesian analysis of two related multi-population mortality models of log-bilinear type, designed for two or more populations. Using a larger set of data, multi-population mortality models allow joint modelling and projection of mortality rates by identifying characteristics shared by all sub-populations as well as sub-population specific effects on mortality. This is important when modeling and forecasting mortality of males and females, regions within a country and when dealing with index-based longevity hedges. Our first model is inspired by the two factor Lee-Carter model of *A. E. Renshaw* and *S. Haberman* [Insur. Math. Econ. 33, No. 2, 255–272 (2003; Zbl 1103.91371)] and the common factor model of *L. R. Carter* and *R. D. Lee* ["Modeling and forecasting US sex differentials in mortality", Int. J. Forecast. 8, No. 3, 393–411 (1992; doi:10.1016/0169-2070(92)90055-E)]. The second model is the augmented common factor model of *N. Li* and *R. Lee* ["Coherent mortality forecasts for a group of populations: an extension of the Lee-Carter method", Demography 42, No. 3, 575–594 (2005; doi:10.1353/dem.2005.0021)]. This paper approaches both models in a statistical way, using a Poisson distribution for the number of deaths at a certain age and in a certain time period. Moreover, we use Bayesian statistics to calibrate the models and to produce mortality forecasts. We develop the technicalities necessary for Markov Chain Monte Carlo (MCMC) simulations and provide software implementation (in R) for the models discussed in the paper. Key benefits of this approach are multiple. We jointly calibrate the Poisson likelihood for the number of deaths and the times series models imposed on the time dependent parameters, we enable full allowance for parameter uncertainty and we are able to handle missing data as well as small sample populations. We compare and contrast results from both models to the results obtained with a frequentist single population approach and a least squares estimation of the augmented common factor model.

MSC:

91D20	Mathematical geography and demography
91B30	Risk theory, insurance (MSC2010)
62F15	Bayesian inference
62P05	Applications of statistics to actuarial sciences and financial mathematics
91B84	Economic time series analysis
91G60	Numerical methods (including Monte Carlo methods)

Keywords:

projected life tables; multi-population stochastic mortality models; Bayesian statistics; Poisson regression; one factor Lee-Carter model; two factor Lee-Carter model; Li-Lee model; augmented common factor model

Software:

BayesDA; R; GMRFLib

Full Text: DOI Link

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