The Pell numbers $P_n$ are defined recursively by $P_1 = 1$, $P_2 = 2$, and $P_n = 2P_{n-1} + P_{n-2}$. The Pell-Lucas numbers $Q_n$ satisfy the same recurrence, but with initial values $Q_1 = 1$ and $Q_2 = 3$. The relation between these sequences is similar to that between Fibonacci and Lucas numbers. They occur in the study of Pell’s equation $x^2 - dy^2 = (-1)^n$ with $d = 2$: the solutions of this equation are $(Q_n, P_n)$.

Similarly to the book on “Fibonacci and Lucas numbers with applications” by the same author [New York, NY: Wiley (2001; Zbl 0984.11010)], the book under review contains a comprehensive overview of properties of Pell and Pell-Lucas numbers and their relations with a wealth of other mathematical objects. The book is divided into 20 chapters.

First, the mathematical concepts used in the book are recalled, in particular linear recurrences, generating functions, binomial coefficients, matrices and determinants. The solutions of Pell’s equation are discussed in Chapters 2 and 3, first without using continued fractions, then the proofs are simplified with the help of the theory of continued fractions. The remaining chapters deal with relations to Pythagorean triples, triangular numbers, Pascal’s triangle and Pell triangles, lattice walks, Chebyshev polynomials and Pell polynomials, trigonometry, graphs, tilings, etc. Similarly to the Fibonacci numbers, Pell numbers also satisfy a large number of identities, and there also exist identities relating Pell and Fibonacci numbers.

The problems and solutions are mostly taken from the journals Fibonacci Quarterly, Mathematical Gazette, Mathematical Spectrum, Mathematics Magazine, and American Mathematical Monthly. The book contains many examples and most chapters end with a list of exercises, which makes the book particularly appropriate for undergraduate/graduate students.

Reviewer: Wolfgang Steiner (Sydney)

MSC:

11-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to number theory
11B39 Fibonacci and Lucas numbers and polynomials and generalizations
11D09 Quadratic and bilinear Diophantine equations

Keywords:
Pell equation; Pell number; Fibonacci number; continued fractions; triangular number; Pythagorean triple; Pascal’s triangle; Chebyshev polynomial; lattice walk

Full Text: DOI