Morrison, Ralph
Tropical images of intersection points. (English) Zbl 1331.14059
Collect. Math. 66, No. 2, 273-283 (2015).

Let K be an algebraically closed non-Archimedean field with a nontrivial valuation $K^* \to \mathbb{R}$. The main example is the field of Puiseux series with complex coefficients. Consider two algebraic curves X and Y in the algebraic torus $(K^*)^2$. Assume that X and Y intersect in only finitely many points. The coordinatewise valuation map applied to the intersection points (with multiplicity) yields a divisor D inside the intersection $\text{Trop}(X) \cap \text{Trop}(Y)$ of the tropicalizations of the two curves.

The main result of the paper under review is that D is linearly equivalent on $\text{Trop}(X)$ to the stable intersection divisor E of $\text{Trop}(X)$ and $\text{Trop}(Y)$. I.e., $D - E$ is the divisor associated to a suitable tropical rational function on $\text{Trop}(X)$. Two proofs are given, one based on tropical modifications and one on Berkovich analytic spaces, although the latter proof requires an additional assumption on X.

The author conjectures that his result is the only restriction on D (in a suitable sense). Detailed examples are given as evidence supporting this conjecture.

Reviewer: Péter E. Frenkel (Budapest)

MSC:

14T05	Tropical geometry (MSC2010)
14C17	Intersection theory, characteristic classes, intersection multiplicities in algebraic geometry
14H50	Plane and space curves

Cited in **4** Documents

Keywords:

tropical curve; stable intersection divisor; linear equivalence; tropical modification; Berkovich analytic space

Full Text: DOI arXiv

References:

[1] Baker, M; Norine, S, Riemann-Roch and Abel-Jacobi theory on a finite graph, Adv. Math., 215, 766-788, (2007) · Zbl 1124.05049 · doi:10.1016/j.aim.2007.04.012

[2] Baker, M., Payne, S., Rabinoff, J.: Nonarchimedean geometry, tropicalization, and metrics on curves (2012). Preprint: http://arxiv.org/abs/1104.0320 · Zbl 06609386

[3] Brugallé, EA; López de Medrano, LM, Inflection points of real and tropical plane curves, J. Singul., 4, 74-103, (2012) · Zbl 1292.14042

[4] Gathmann, A; Kerber, M, A Riemann-Roch theorem in tropical geometry, Mathematische Zeitschrift, 259, 217-230, (2008) · Zbl 1187.14066 · doi:10.1007/s00209-007-0222-4

[5] Gubler, W.: A guide to tropicalizations. In: Algebraic and combinatorial aspects of tropical geometry. Contemp. Math., , vol. 589, pp. 125-189. Amer. Math. Soc., Providence (2013) · Zbl 1318.14061

[6] Hasse, C; Musiker, G; Yu, J, Linear systems on tropical curves, Mathematische Zeitschrift, 270, 1111-1140, (2012) · Zbl 1408.14201 · doi:10.1007/s00209-011-0844-4

[7] Luo, Y.: Tropical Convexity and Canonical Projections (2013). Preprint: http://arxiv.org/abs/1304.7963 · Zbl 1124.05049

[8] Maclagan, D., Sturmfels, B.: Introduction to tropical geometry (2014). Preprint: http://homepages.warwick.ac.uk/staff/D.Maclagan/papers/Tropic · Zbl 1321.14048

[9] Mikhalkin, G.: Tropical Geometry and its applications. In: International Congress of Mathematicians, vol. II, pp. 827-852. Eur. Math. Soc., Zürich (2006) · Zbl 1103.14034

[10] Osserman, B; Payne, S, Lifting tropical intersections, Doc. Math., 18, 121-175, (2013) · Zbl 1308.14069

[11] Osserman, B.; Rabinoff J.: Lifting non-proper tropical intersections. Contemp. Math. (2014, to appear) · Zbl 1320.14078

paper as accurately as possible without claiming the completeness or perfect precision of the matching.