Desvillettes, L.; Lepoutre, T.; Moussa, A.; Trescases, A.
On the entropic structure of reaction-cross diffusion systems. (English)

This article is concerned with the existence analysis of reaction-(cross-)diffusion systems of the form

$$\partial_t U - \Delta [A(U)] = R(U)$$

for a vector-valued unknown $U = (u_1, \ldots, u_I)$ such that its components be nonnegative-valued and satisfy
homogeneous Neumann boundary conditions on a bounded domain $\Omega \subset \mathbb{R}^d$.

In addition to certain continuity properties and bounds of the parameter functions $A$ and $R$, the central assumption on $A$ is that it is a homeomorphism from the nonnegative cone $[0, \infty)^I$ to itself. The cornerstone of this article is a novel semi-discretization scheme reading as

$$\frac{U^k - U^{k-1}}{\tau} - \Delta [A(U^k)] = R(U^k) \quad \text{in } \Omega, \quad \partial_n A(U^k) = 0 \quad \text{on } \partial \Omega.$$

The well-posedness of the scheme is proved by a fixed-point argument. The key feature of this discretization is the preservation of an entropic structure of the PDE system at hand (existing in the considered examples) which itself can be of use for finding the sufficient a priori estimates to obtain a weak solution after passing to the continuous-time limit $\tau \to 0$.

These ideas are, amongst others, applied to cross-diffusion systems already considered in [L. Desvillettes et al., SIAM J. Math. Anal. 46, No. 1, 820–853 (2014; Zbl 1293.35142)], now allowing for a wider parameter range.

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MSC:
35K57 Reaction-diffusion equations
35K51 Initial-boundary value problems for second-order parabolic systems
35K55 Nonlinear parabolic equations
35Q92 PDEs in connection with biology, chemistry and other natural sciences
92D25 Population dynamics (general)

Keywords:
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References:
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[8] DOI: 10.1007/978-3-642-61798-0 · Zbl 0695.35060 · doi:10.1007/978-3-642-61798-0

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