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Fourier pseudospectral method with Kepler mapping for travelling waves with discontinuous slope: application to corner waves of the Ostrovsky-Hunter equation and equatorial Kelvin waves in the four-mode approximation. (English) [Zbl 1331.76082]


Summary: Many species of travelling waves have a single branch of solutions which ends at finite amplitude with a singular wave whose slope is discontinuous, a so-called “corner wave”. Fourier pseudospectral methods converge exponentially fast with $N$, the truncation of the series, for the smooth waves, but the error falls only as $O(1/N)$ for a wave with a slope discontinuity. We show that the error rate can be accelerated to $O(1/N^3)$ by making the change-of-coordinate (“Kepler map”) $x = z - \sin(z)$. Unfortunately, there is a subtlety: the end-in-a-singular-solution bifurcation is possible only for infinite-dimensional systems. We show that the truncated Fourier pseudospectral approximation has roots on both sides of the corner wave. The bifurcation can be detected by (i) a step-function-like jump in the residual of the differential equation at the corner wave and (ii) by observing the slope of $u(x)$, which becomes discontinuous only at the corner wave itself. These concepts are illustrated for the equatorially trapped ocean Kelvin wave in the so-called “four-latitudinal-mode” approximation. However, corner waves arise in many species of waves and the concepts explained here are applicable to all.

MSC:

76M15 Boundary element methods applied to problems in fluid mechanics
65M70 Spectral, collocation and related methods for initial value and initial-boundary value problems involving PDEs
76M22 Spectral methods applied to problems in fluid mechanics

Full Text: DOI

References:


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