Anzaldo-Meneses, A.

On non-autonomous dynamical systems. (English) Zbl 1333.37069


In this work, a class of classical systems with time-dependent nonlinear Hamiltonians is analyzed. One of the main difficulties is that the Hamiltonian is time-dependent and therefore not preserved.

This fact makes it natural to search for invariants, which somehow help to understand or to describe the time variation of the Hamiltonian. Sometimes, it may be very useful to have knowledge of symmetries by applying Noether’s Theorem or the usage of canonical transformations.

Alternatively, there are general methods to study this problem analytically. Here, a new simple method to study nonlinear, non-autonomous classical problems is proposed based on the introduction of a set of coordinates, in general infinite, and leading to equivalent linear problems. It is then suggested to formally solve the resulting linear problem and to use the initial conditions to construct invariants.

Two examples for flat metrics are given. In the first example, a nonlinear time-dependent one-dimensional case is solved in terms of elliptic functions using the proposed method. It is shown that the series expansion of the addition theorem for elliptic functions corresponds to the expected infinite linear combination in terms of a basis of nonlinear coordinates. Elliptic functions are common in many solvable problems in modern classical mechanics since its beginnings. A linear invariant surface is constructed explicitly.

The second example is a harmonic oscillator with three time-dependent parameters and its geometry is naturally described by two sets of coordinates in the parameter space and in a space formed by ad-hoc quadratic coordinates defining a cone and constructed from the two-dimensional phase space coordinates. The Poisson algebra satisfied by the quadratic coordinates is then a symplectic algebra and the trajectories are described in the three-dimensional space. It is natural to see the trajectory in quadratic coordinates as lying on a “light” cone defined as the set of null-vectors.

From the abstract: “It is shown further that the quantum mechanical problem of scattering in a superlattice leads to mathematically equivalent equations for the wave function, if the classical time is replaced by the space coordinate along a superlattice. The mathematical method used to compute the trajectories for stepwise constant parameters can be applied to both problems. It is the standard method in quantum scattering calculations, as known for locally periodic systems including a space-dependent effective mass.”

Reviewer: Irina V. Konopleva (Ulyanovsk)

MSC:
37J05 Relations of dynamical systems with symplectic geometry and topology (MSC2010)
37J15 Symmetries, invariants, invariant manifolds, momentum maps, reduction (MSC2010)
37J25 Stability problems for finite-dimensional Hamiltonian and Lagrangian systems
58A20 Jets in global analysis
37N20 Dynamical systems in other branches of physics (quantum mechanics, general relativity, laser physics)
34L25 Scattering theory, inverse scattering involving ordinary differential operators
81U05 2-body potential quantum scattering theory

Keywords:
time dependent nonlinear Hamiltonians; invariants symmetry usage via Noether’s theorem; alternative analytical approach; nonlinear and quadratic Hamiltonians

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References:

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