Filmus, Yuval; **Lauria, Massimo**; **Mikša, Mladen**; **Nordström, Jakob**; **Vinyals, Marc**

Towards an understanding of polynomial calculus: new separations and lower bounds (extended abstract). (English) Zbl 1336.03065

Fomin, Fedor V. (ed.) et al., Automata, languages, and programming. 40th international colloquium, ICALP 2013, Riga, Latvia, July 8–12, 2013, Proceedings, Part I. Berlin: Springer (ISBN 978-3-642-39205-4/pbk). Lecture Notes in Computer Science 7965, 437-448 (2013).

Summary: During the last decade, an active line of research in proof complexity has been into the space complexity of proofs and how space is related to other measures. By now these aspects of resolution are fairly well understood, but many open problems remain for the related but stronger polynomial calculus (PC/PCR) proof system. For instance, the space complexity of many standard "benchmark formulas" is still open, as well as the relation of space to size and degree in PC/PCR.

We prove that if a formula requires large resolution width, then making XOR substitution yields a formula requiring large PCR space, providing some circumstantial evidence that degree might be a lower bound for space. More importantly, this immediately yields formulas that are very hard for space but very easy for size, exhibiting a size-space separation similar to what is known for resolution. Using related ideas, we show that if a graph has good expansion and in addition its edge set can be partitioned into short cycles, then the Tseitin formula over this graph requires large PCR space. In particular, Tseitin formulas over random 4-regular graphs almost surely require space at least $\Omega(\sqrt{n})$.

Our proofs use techniques recently introduced in [*I. Bonacina* and *N. Galesi*, "Pseudo-partitions, transversality and locality: a combinatorial characterization for the space measure in algebraic proof systems", in: Proceedings of the 4th conference on innovations in theoretical computer science, ITCS'13. New York, NY: Association for Computing Machinery (ACM). 455–472 (2013; doi:10.1145/2422436.2422486)]. Our final contribution, however, is to show that these techniques provably cannot yield non-constant space lower bounds for the functional pigeonhole principle, delineating the limitations of this framework and suggesting that we are still far from characterizing PC/PCR space.

For the entire collection see [Zbl 1268.68018].

MSC:

03F20	Complexity of proofs
68Q17	Computational difficulty of problems (lower bounds, completeness, difficulty of approximation, etc.)

Full Text: DOI