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Linear stability of algebraic Ricci solitons. (English) [Zbl 1337.53055]

Summary: We consider a modified Ricci flow equation whose stationary solutions include Einstein and Ricci soliton metrics, and we study the linear stability of those solutions relative to the flow. After deriving various criteria that imply linear stability, we turn our attention to left-invariant soliton metrics on (non-compact) simply connected solvable Lie groups and prove linear stability of many such metrics. These include an open set of two-step solvsolitons, all two-step nilsolitons, two infinite families of three-step solvable Einstein metrics, all nilsolitons of dimensions six or less, and all solvable Einstein metrics of dimension seven or less with codimension-one nilradical. For each linearly stable metric, dynamical stability follows from a generalization of the techniques of Guenther, Isenberg, and Knopf.

MSC:
53C25 Special Riemannian manifolds (Einstein, Sasakian, etc.)
37L05 General theory of infinite-dimensional dissipative dynamical systems, nonlinear semigroups, evolution equations
53C21 Methods of global Riemannian geometry, including PDE methods; curvature restrictions
35K55 Nonlinear parabolic equations
22E25 Nilpotent and solvable Lie groups
53C30 Differential geometry of homogeneous manifolds

Keywords: linear stability; left-invariant soliton metrics; two-step solvsolitons; two-step nilsolitons; dynamical stability

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