The authors consider the following problem, first introduced by Y. Caro and R. Yuster [Discrete Math. 311, No. 16, 1786–1792 (2011; Zbl 1223.05065)]. An MC-coloring of a graph $G$ is a (not necessarily proper) edge coloring of $G$ such that for every pair of vertices $u, v \in V(G)$, there is a monochromatic $u, v$-path. The monochromatic connection number of $G$, denoted $mc(G)$, is the maximum number of colors in an MC-coloring of $G$. Caro and Yuster [loc. cit.] proved that $mc(G) \geq |E(G)| - |V(G)| + 2$ for every graph $G$ and gave several different sufficient conditions under which equality holds.

The authors prove upper and lower bounds on $mc(G \star H)$, where $G \star H$ is the lexicographic, strong, Cartesian, or direct product of the graphs $G$ and $H$. They also provide sharpness examples for the lower bounds.

The lower bounds are mainly based on the inequality $mc(G) \geq |E(G)| + |V(G)| + 2$, and the upper bounds are mainly based on the inequality $mc(G) \leq |E(G)| - |V(G)| + \kappa(G) + 1$, both of which are due to [loc. cit.]. The sharpness examples are based on the sufficient conditions of Caro and Yuster [loc. cit.] mentioned above.

In Section 3, the authors apply their results to several specific choices of $G$ and $H$. There are some unsubstantiated claims in this section: in particular, the authors claim in Proposition 2 that the diameter of $P_L \circ \cdots \circ P_L$ is at least 3, where $\circ$ is the lexicographic product, but when all $L_i \in \{1, 2\}$, this product is a clique and thus has diameter 1. Similarly, in Proposition 3, the authors claim that the diameter of $R_i$ is at least 3 for $i \geq 3$, where $R_i$ is the cycle on $i$ vertices, but this is false when $i \in \{3, 4, 5\}$.

Reviewer: Gregory J. Puleo (Urbana)

MSC:
05C40 Connectivity
05C15 Coloring of graphs and hypergraphs
05C76 Graph operations (line graphs, products, etc.)

Keywords:
monochromatic path; monochromatic connection; graph products

Full Text: DOI arXiv

References:


[4] Q. Cai, X. Li and D. Wu, Erdős-Gallai-type results for colorful monochromatic connectivity of a graph, accepted by J. Combin. Optim. · Zbl 1358.05097


19. X. Li, Y. Shi and Y. Sun, Rainbow connections of graphs: A survey, Graphs Combin.29 (2013) 1-38. \(\text{genRefLink}(16, 'S1793830916500117BIB019', '10.1007%252F252F00373-012-1243-2');\)


25. X. Zhu, Game coloring the Cartesian product of graphs, J. Graph Theory59 (2008) 261-278. \(\text{genRefLink}(16, 'S1793830916500117BIB025', '10.1002%252Fjgt.20338');\)

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.