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Geometrical and analytical properties of Chebyshev sets in Riemannian manifolds. (English)

The closed subsets of Riemannian manifolds known as Chebyshev sets are characterized by the existence of a well-defined distance-realizing projection onto them. Chebyshev sets and their relation with convexity have long been considered in the context of Banach and Hilbert spaces. A development of this theory in the context of Riemannian manifolds needs some adaptions. First, one must consider not only many different concepts of convexity, but also Chebyshev sets, called simple, satisfying the condition that the minimizing geodesic realizing distance to it are unique.

The author characterizes simple Chebyshev sets as suns (a special type of a closed set $C$ of a complete Riemannian manifold $M$). Moreover, a simple Chebyshev set of a complete connected Riemannian manifold $M$ of nonnegative curvature is totally convex. He shows that a simple Chebyshev set of $M$ with empty boundary is a submanifold of $M$ whose normal bundle is diffeomorphic to $M$. Finally, the main result is that in a complete connected Riemannian manifold $M$ of nonnegative Ricci curvature, the distance function to a simple Chebyshev set is (strongly) subharmonic. Conversely, if the sectional curvature of $M$ is nonnegative and the distance function to a closed set $C \subset M$ is (strongly) subharmonic, then $C$ is a simple Chebyshev set.

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References: