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Tree Nash equilibria in the network creation game. (English) Zbl 1342.05169

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Summary: In the network creation game with n vertices, every vertex (a player) buys a set of adjacent edges, each at a fixed amount $\alpha > 0$. It has been conjectured that for $\alpha \geq n$, every Nash equilibrium is a tree, and has been confirmed for every $\alpha \geq 273 \cdot n$. We improve upon this bound and show that this is true for every $\alpha \geq 65 \cdot n$. To show this, we provide new and improved results on the local structure of Nash equilibria. Technically, we show that if there is a cycle in a Nash equilibrium, then $\alpha < 65 \cdot n$. Proving this, we only consider relatively simple strategy changes of the players involved in the cycle. We further show that this simple approach cannot be used to show the desired upper bound $\alpha < n$ (for which a cycle may exist), but conjecture that a slightly worse bound $\alpha < 1.3 \cdot n$ can be achieved with this approach. Towards this conjecture, we show that if a Nash equilibrium has a cycle of length at most 10, then indeed $\alpha < 1.3 \cdot n$. We further provide experimental evidence suggesting that when the girth of a Nash equilibrium is increasing, the upper bound on α obtained by the simple strategy changes is not increasing. To the end, we investigate the approach for a coalitional variant of Nash equilibrium, where coalitions of two players cannot collectively improve, and show that if $\alpha \geq 41 \cdot n$, then every such Nash equilibrium is a tree.

For the entire collection see [[Zbl 1298.68022](#)].

MSC:

- [05C82](#) Small world graphs, complex networks (graph-theoretic aspects)
- [05C57](#) Games on graphs (graph-theoretic aspects)
- [91A43](#) Games involving graphs
- [91A10](#) Noncooperative games
- [68M10](#) Network design and communication in computer systems

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