Lieblich, Max; Olsson, Martin

The bounded derived category $D(X)$ of coherent sheaves has become by now a standard invariant for studying a smooth projective variety $X$. It is particular useful for revealing “hidden” relationships between algebraic varieties.

For varieties $X,Y$ over $\mathbb{C}$, derived equivalences $D(X) \cong D(Y)$ have been shown in many cases to correspond to equivalences of other transcendental invariants that are more easily studied. For example, Orlov’s work on $K3$ surfaces in [D. O. Orlov, J. Math. Sci., New York 84, No. 5, 1361–1381 (1997; Zbl 0938.14019)] shows that two $K3$ surfaces are derived equivalent if and only if there is a Hodge isometry between their respective transcendental lattices.

The paper under review extends to positive characteristic Orlov’s classical result for complex $K3$ surfaces. In particular, the authors show that if $X$ is a $K3$ surface over an algebraically closed field $k$ of positive characteristic $\neq 2$, then any smooth projective variety $Y$ over $k$ that is derived equivalent to $X$ must be a $K3$ surface isomorphic to a fine moduli space $M_X(v)$ of stable sheaves on $X$ of Mukai vector $v$. Similarly, the authors determine that $X$ admits only finitely many so-called Fourier-Mukai partners $Y$, and they show further that if the Picard rank of $X$ is at least 11, then $Y \cong X$. The main tool in the proof is a derived category version of the Torelli theorem, removing the characteristic 0 restriction.

In motivic fashion, the authors also relate derived equivalences to other cohomological realizations (étale, crystalline, de Rham, Chow), and they use this to prove, for example, that derived equivalent $K3$ surfaces have the same zeta-function and to establish the variational crystalline Hodge conjecture in some special cases.

Reviewer: Howard Nuer (Piscataway)

MSC:
14F05 Sheaves, derived categories of sheaves, etc. (MSC2010)
14J28 $K3$ surfaces and Enriques surfaces
14G17 Positive characteristic ground fields in algebraic geometry
14C30 Transcendental methods, Hodge theory (algebro-geometric aspects)
14F30 $p$-adic cohomology, crystalline cohomology

Keywords:
Fourier-Mukai equivalence; $K3$ surfaces; zeta function; motives

Full Text: DOI Link arXiv