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Critical points and number of master integrals. (English) Zbl 1342.81139
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Summary: We consider the question about the number of master integrals for a multiloop Feynman diagram. We show that, for a given set of denominators, this number is totally determined by the critical points of the polynomials entering either of the two representations: the parametric representation and the Baikov representation. In particular, for the parametric representation the corresponding polynomial is just the sum of Symanzik polynomials. The relevant topological invariant is the sum of the Milnor numbers of the proper critical points. We present a Mathematica package Mint to automatize the counting of the master integrals for the typical case, when all critical points are isolated.

MSC:

81Q30 Feynman integrals and graphs; applications of algebraic topology and algebraic geometry Cited in **25** Documents

Keywords:

scattering amplitudes; differential geometry; algebraic geometry

Software:

LiteRed; SageMath; Mint ; Mathematica; Reduze

Full Text: [DOI](#) [arXiv](#)

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