Let $G$ be a finite group. A finite group $X$ is called a Frattini cover of $G$ if $X/F = G$, where $F$ is contained in the Frattini subgroup $\Phi(X)$ of $X$. Frattini covers have been studied considerably with respect to coverings of curves. Also, Frattini covers come up in studying minimal counterexamples.

The first main result of this paper is a lifting theorem for Frattini covers $X$ of $G = X/F$ with $F$ a primary group of odd order. Using this result, J. G. Thompson’s classification of the finite simple groups in which every proper subgroup is solvable [Bull. Am. Math. Soc. 74, 383-437 (1968; Zbl 0159.30804)] and a result of I. M. Isaacs [Am. J. Math. 95, 594-635 (1973; Zbl 0277.20008)], the authors obtain the following characterization of finite solvable groups (M. J. J. Barry [in “On conditions related to nonsolvability”, arXiv:1109.4913 (2011)] asked whether this was true): a finite group $G$ is solvable if and only if $x_1x_2x_3 \neq 1$ for all nontrivial $p_i$-elements $x_i$ of $G$ for distinct primes $p_i$, $i = 1, 2, 3$. Thompson [loc. cit.] proved this result if one considers all triples of nontrivial elements of coprime order.

In addition, the authors obtain some other characterizations of finite solvable groups, give a short proof of a theorem of W. Feit and J. Tits [Can. J. Math. 30, 1092-1102 (1978; Zbl 0358.20014)] about the minimal dimension of a representation of a group which has a section isomorphic to a given simple group, and discuss some connections between lifting theorems and coverings of curves over $C$.

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MSC:

20D10 Finite solvable groups, theory of formations, Schunck classes, Fitting classes, $\pi$-length, ranks
20E22 Extensions, wreath products, and other compositions of groups
20D25 Special subgroups (Frattini, Fitting, etc.)
20C15 Ordinary representations and characters
14H30 Coverings of curves, fundamental group

Keywords:
finite groups; Frattini covers; characterizations of finite solvable groups; representations of simple groups; coverings of curves

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