Under consideration is the problem

\[-\Delta u = v^{p-1}, \quad -\Delta v = \rho(x)u^{p-1} + f(x, u) \quad (x \in \mathbb{R}^n), \quad u, v \to 0 \text{ as } |x| \to \infty,\]

where \(\rho\) is a positive function. This system is reduced to an equation of fourth order (if we find \(v\) from the former equation and insert it into the latter) and under certain natural conditions on the functions \(\rho, f(x, u)\) the existence of weak solutions is proven by a variational method. The corresponding functional whose critical points are solutions is written as

\[
I(u) = \frac{p-1}{p} \int_{\mathbb{R}^n} |\Delta u|^{p-1} + \rho(x)|u|^{p-1} \, dx - \int_{\mathbb{R}^n} F(x, u) \, dx, \quad F(x, u) = \int_0^u f(\tau, u) \, d\tau.
\]

The tool is the generalized Weierstrass theorem and a variant of the symmetric mountain pass theorem. Under some additional conditions, multiplicity results are obtained, i.e., the existence of a countable set of weak solutions with certain properties is established.

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MSC:

\begin{itemize}
  \item 35J35 Variational methods for higher-order elliptic equations
  \item 35J50 Variational methods for elliptic systems
  \item 35J58 Boundary value problems for higher-order elliptic systems
  \item 35J60 Nonlinear elliptic equations
  \item 35D30 Weak solutions to PDEs
\end{itemize}

Keywords:

nonlinear elliptic system of Lane-Emden type; subquadratic growth; fourth order elliptic equation; variational method; compact imbedding

Full Text: DOI

References:

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