Adarbeh, K.; Kabbaj, S.
Zaks’ conjecture on rings with semi-regular proper homomorphic images. (English) 

A commutative ring \( R \) is called semi-regular if each injective \( R \)-module is flat. Each semi-regular ring is coherent, i.e. every finitely generated ideal is finitely presented. The authors define \( R \) to be residually semi-regular (resp. coherent) if \( R/I \) is semi-regular (resp. coherent) for each finitely generated \( I \). It is proven that \( R \) is residually semi-regular if and only if \( R \) satisfies one of the following conditions:

1. \( R \) is a local ring of maximal ideal \( P \) satisfying \( P^2 = 0 \) and \( \dim_{R/P} P \leq 2 \);
2. \( R \) is residually coherent and arithmetical (i.e. its lattice of ideals is distributive).

When \( R \) is reduced it is shown that \( R \) is residually semi-regular if and only if \( R \) is either a Prüfer (arithmetical) domain or a von Neumann regular ring.

So, the main result of this paper is a generalization of the following:

1. A Noetherian ring \( R \) verifies that that \( R/I \) is self injective if and only if either \( R \) satisfies 1 or \( R \) is an Artinian arithmetical ring or \( R \) is a Dedekind domain [L. S. Levy, Pac. J. Math. 18, 149-153 (1966; Zbl 0139.26403)];
2. an integral domain is residually semi-regular if and only if it is Prüfer (Zak’s conjecture) [E. Matlis, J. Algebra 95, 343-372 (1985; Zbl 0596.13014)].

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MSC:
13C10 Projective and free modules and ideals in commutative rings
13C11 Injective and flat modules and ideals in commutative rings
13E05 Commutative Noetherian rings and modules
13F05 Dedekind, Prüfer, Krull and Mori rings and their generalizations
13H10 Special types (Cohen-Macaulay, Gorenstein, Buchsbaum, etc.)

Keywords:
semi-regular ring; IF-ring; coherent ring; arithmetical ring; quasi-Frobenius ring; self fp-injective ring; Prüfer domain; Dedekind domain

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References:
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