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**Minimization of the Perron eigenvalue of incomplete pairwise comparison matrices by Newton iteration.** (English) [Zbl 1349.91092](#)

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Summary: Pairwise comparison matrices are of key importance in multi-attribute decision analysis. A matrix is incomplete if some of the elements are missing. The eigenvector method, to derive the weights of criteria, can be generalized for the incomplete case by using the least inconsistent completion of the matrix. If inconsistency is indexed by CR, defined by Saaty, it leads to the minimization of the Perron eigenvalue. This problem can be transformed to a convex optimization problem. The paper presents a method based on the Newton iteration, univariate and multivariate. Numerical examples are also given.

**MSC:**

[91B06](#) Decision theory  
[49M15](#) Newton-type methods  
[90B50](#) Management decision making, including multiple objectives

**Keywords:**

[incomplete pairwise comparison matrix](#); [Perron eigenvalue](#); [Newton iteration](#)

**Software:**

[fminbnd](#); [BRENT](#); [Matlab](#)

**Full Text:** [DOI](#)

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