Balogh, József; Treglown, Andrew; Wagner, Adam Zsolt

Applications of graph containers in the Boolean lattice. (English)


Summary: We apply the graph container method to prove a number of counting results for the Boolean lattice \( P(n) \). In particular, we:

(i) Give a partial answer to a question of A. Sapozhenko [Lect. Notes Comput. Sci. 3777, 1–13 (2005; Zbl 1159.68656)] estimating the number of \( t \) error correcting codes in \( P(n) \), and we also give an upper bound on the number of transportation codes;

(ii) Provide an alternative proof of Kleitman’s theorem on the number of antichains in \( P(n) \) and give a two-coloured analogue;

(iii) Give an asymptotic formula for the number of \((p, q)\)-tilted Sperner families in \( P(n) \);

(iv) Prove a random version of Katona’s \( t \)-intersection theorem.

In each case, to apply the container method, we first prove corresponding supersaturation results. We also give a construction which disproves two conjectures of L. Ilinca and J. Kahn [Order 30, No. 2, 427–435 (2013; Zbl 1297.06005)] on maximal independent sets and antichains in the Boolean lattice. A number of open questions are also given.

MSC:

05C69 Vertex subsets with special properties (dominating sets, independent sets, cliques, etc.)

05C65 Hypergraphs

05C30 Enumeration in graph theory

06A07 Combinatorics of partially ordered sets

Keywords:

container method; Boolean lattice; Sperner families; error correcting codes; enumeration problems

Full Text: DOI arXiv Link

References:


