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The local symbol complex of a reciprocity functor. (English) Zbl 1353.19003

Ann. K-Theory 1, No. 3, 317-338 (2016).

A theory of *reciprocity functors* and associated K-groups has recently been developed by *F. Ivorra* and *K. Rülling* ["K-groups of reciprocity functors", Preprint, to appear in J. Alg. Geom., arxiv:1209.1217], building on ideas of B. Kahn and K. Somekawa. Let F be a perfect field. Let $\mathrm{Reg}^{\leq 1}\mathrm{Cor}$ be the category whose objects are regular F-schemes of dimension at most 1 which are separated and of finite type over a field k which is finitely-generated over F. A reciprocity functor $\mathcal{M}$ is a presheaf of abelian groups on $\mathrm{Reg}^{\leq 1}\mathrm{Cor}$ satisfying certain additional properties which guarantee the existence of local symbols associated to points on smooth projective geometrically connected curves over k yielding a generalized reciprocity law for the curve. Examples include smooth commutative algebraic groups over $S = \mathrm{Spec} F$, homotopy invariant Nisnevich sheaves with transfers and Rost's cycle modules.

Given a collection of reciprocity functors $\mathcal{M}_1, \dots, \mathcal{M}_r$, Ivorra and Rülling associate to it a kind of product $T(\mathcal{M}_1, \dots, \mathcal{M}_r)$, also a reciprocity functor, which they call the K-group of $\mathcal{M}_1, \dots, \mathcal{M}_r$. When k is algebraically closed and C is a smooth complete curve over k with generic point η_C a reciprocity functor $\mathcal{M}$, and the associated reciprocity law, gives rise to a complex of abelian groups:

$$\left(\mathcal{M} \otimes^M \mathbb{G}_m\right)(\eta_C) \to \oplus_{P\in C}\mathcal{M}(k) \to \mathcal{M}(k)$$

where $\otimes^M$ is the tensor product of Mackey functors. The main theorem of the article under review (Theorem 3.11) is that the homology of this complex is naturally isomorphic to the K-group $T(\mathcal{M}, \mathrm{CH}_0(C)^0)(k)$ when this latter group satisfies a pair of technical conditions (3.3 and 3.10 in the article). The author shows that the conditions of Theorem 3.11 hold in the following cases: (i) $\mathcal{M} = T(\mathcal{F}_1, \dots, \mathcal{F}_r)$ where $\mathcal{F}_1, \dots, \mathcal{F}_r$ are homotopy invariant Nisnevich sheaves with transfers and (ii) $\mathcal{M} = T(\mathbb{G}_a, \mathcal{M}_1, \dots, \mathcal{M}_r)$ where $\mathcal{M}_1, \dots, \mathcal{M}_r$ are arbitrary reciprocity functors. In a final section the author treats some cases in which the main result extends to a non-algebraically closed field k.

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MSC:

19D45 Higher symbols, Milnor K-theory
14C25 Algebraic cycles

Keywords:

reciprocity functor; Milnor K-group; local symbol

Full Text: DOI arXiv Euclid