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Counting (k, l)-sumsets in groups of a prime order. (English) Zbl 1354.11021

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From the text: A subset A of a group G is called (k, l)-sumset, if $A = kB - lB$ for some $B \subseteq G$, where $kB - lB = x_1 + \ldots + x_k - x_{k+1} - \ldots - x_{k+l} : x_1, \ldots, x_{k+l} \in B$. Upper and lower bounds for the number (k, l)-sumsets in groups of prime order are provided.

Write $SS_{k,l}(\mathbb{Z}_p)$ for the collection of (k, l)-sumsets in $\mathbb{Z}_p$. *B. Green* and *I. Ruzsa* in [Stud. Sci. Math. Hung. 41, No. 3, 285–293 (2004; Zbl 1064.11020)] proved

$$p^2 2^{p/3} \ll |SS_{2,0}(\mathbb{Z}_p)| \leq 2^{p/3+\theta(p)}$$

where $\theta(p)/p \to 0$ as $p \to \infty$ and $\theta(p) \ll p(\log\log p)^{2/3}(\log p)^{-1/9}$.

The aim of this work is to obtain bounds for the number $|SS_{k,l}(\mathbb{Z}_p)|$. We prove

Theorem 1. Let p be a prime number and k, l be nonnegative integers with $k + l \geq 2$. Then there exists a positive constant $C_{k,l}$ such that

$$C_{k,l} 2^{p/(2(k+l)-1)} \leq |SS_{k,l}(\mathbb{Z}_p)| \leq 2^{(p/(k+l+1))+(k+l-2)+o(p)}. \tag{1}$$

MSC:

11B75 Other combinatorial number theory
68R05 Combinatorics in computer science
20D60 Arithmetic and combinatorial problems involving abstract finite groups

Keywords:

characteristic function; sumsets; granular set