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Topics in equisingularity theory. (English) Zbl 1354.14002

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The book is based on a series of lectures on equisingularity theory given by the author about results of Zariski, Greuel, Lê, K. Saito, Teissier and many others related to Zariski's multiplicity conjecture.

Let $f_i : (\mathbb{C}^n, 0) \to (\mathbb{C}, 0)$ be germs of analytic functions, $i = 1, 2$. f_1 and f_2 are called topologically V-equivalent if there exist open neighbourhoods U_1, U_2 of $0 \in \mathbb{C}^n$ and a homeomorphism $\varphi : (U_2, 0) \to (U_1, 0)$ such that f_i is defined on U_i and $\varphi(V(f_2)) \cap U_2 = V(f_1) \cap U_1$.

Zariski's multiplicity conjecture states that f_1 and f_2 being topologically V-equivalent implies that the multiplicities $\operatorname{mult}_0(f_1)$ and $\operatorname{mult}_0(f_2)$ are equal. A family $\{f_t\}$ of germs of functions is weakly topologically V-equisingular if for all sufficiently small t, the function f_t is topologically V-equivalent to f_0, $t \in D$ an open disc in $\mathbb{C}$. The family is called μ-constant if the Milnor number of f_t at 0 is independent on t.

If $n \neq 3$ the family is μ-constant if and only if it is weakly topologically V-equisingular. The conjecture that a μ-constant family is equimultiple is still open, only proved in special cases.

The first chapter presents Ephraim's homology approach to Zariskis conjecture. The second chapter introduces and studies the Lê number replacing the Milnor number for non-isolated singularities. Deformations with constant Milnor number are studied in chapter 3. It is proved that in the case $n \neq 3$ in a μ-constant family $\{f_t\}$ the diffeomorphism type of the Milnor fibration of f_t at 0 is independent on t (for small t) and the family is weakly topologically V-equisingular. Criteria for a family being μ-constant are discussed. As a consequence Zariski's conjecture is proved for quasihomogeneous polynomials.

In chapter 4 deformations with constant Lê number are studied. In chapter 5 equisingular deformations of aligned singularities (introduced by Massey) are investigated.

Reviewer: Gerhard Pfister (Kaiserslautern)

MSC:

14-02	Research exposition (monographs, survey articles) pertaining to algebraic geometry
14B05	Singularities in algebraic geometry
14B07	Deformations of singularities
14E15	Global theory and resolution of singularities (algebro-geometric aspects)
32S15	Equisingularity (topological and analytic)
32S55	Milnor fibration; relations with knot theory

Keywords:

Zariski's multiplicity conjecture; topological V-equivalent; topologically equisingular; Le number; aligned singularities